American Community Schools
Department of Mathematics

Every year the math department (all teachers JK-12) prepares revision questions for all grade levels in the Elementary School and all courses in the Middle School and the Academy.

Although it is important for students to rest in the summer months, we encourage our students to spend stimulating and productive time working on the summer practice.

Solutions are provided so that students may assess themselves along the way. The material attached consists of problems relating to this year’s taught curriculum.

Students will be expected to show their summer practice problems in the beginning of the next school year.

The math department (JK-12) wishes everyone a restful and enjoyable summer.

Best regards,

Dr. Tsokos –Division Chair

Ms. Andrikopoulos- Mathematics Coordinator

To prepare for study in IB Math SL Year 2 2018-2019

Paper 1 Practice

Calculator not allowed in this section

1. Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$.

(2)

(b) Write down $g^{-1}(x)$.

(1)

(c) Find $(f \circ g^{-1})(5)$.

(2)
(Total 5 marks)

2. Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

(a) Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln x}{x^3}$.

(4)

(b) The graph of g has a maximum point at A. Find the x-coordinate of A.

(3)
(Total 7 marks)

3. Solve the equation $2\cos x = \sin 2x$, for $0 \le x \le 3\pi$.

(Total 7 marks)

4. Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k.

(5)

(b) The line $y = p$ intersects the graph of f. Find all possible values of p.

(2)
(Total 7 marks)

5. In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

(a) Find d.

(2)

(b) Find u_{20}.

(2)

(c) Find S_{20}.

(2)
(Total 6 marks)

6. Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

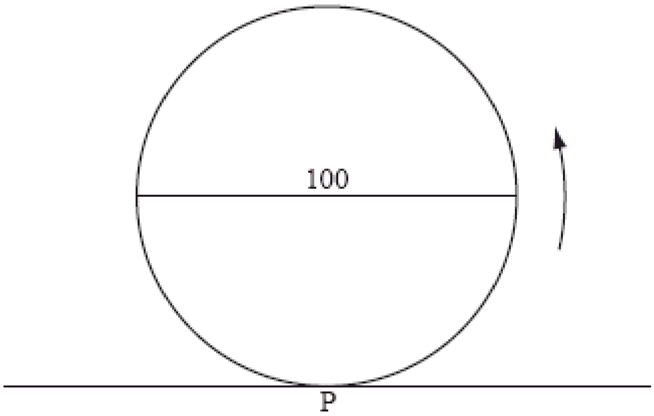
(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$.

(4)

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation.

(3)
(Total 7 marks)

7. The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after

(i) 10 minutes;

(ii) 15 minutes.

(2)

Let $h(t)$ metres be the height of P above ground level after t minutes. Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

(b) (i) Show that $h(8) = 90.5$.

(ii) Find $h(21)$.

(4)

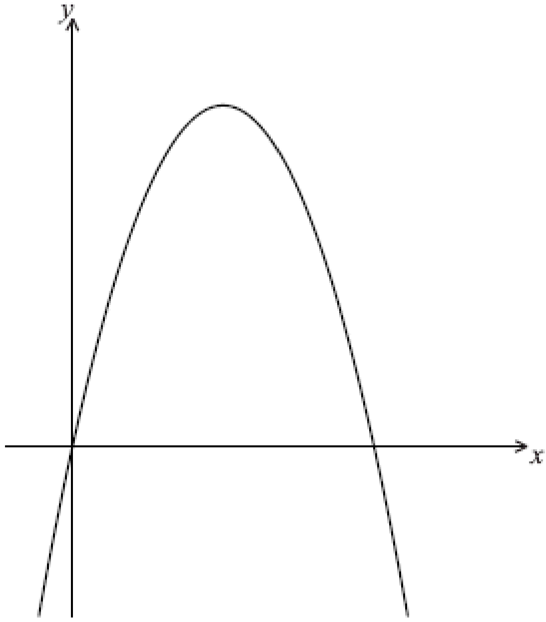
(c) **Sketch** the graph of h, for $0 \le t \le 40$.

(3)

(d) Given that h can be expressed in the form $h(t) = a \cos bt + c$, find a, b and c.

(5)
(Total 14 marks)

8. Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



(a) Find the x-intercepts of the graph.

(4)

(b) (i) Write down the equation of the axis of symmetry.

(ii) Find the y-coordinate of the vertex.

(3)
(Total 7 marks)

10. The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

(a) Write down the value of $\tan \theta$.

(1)

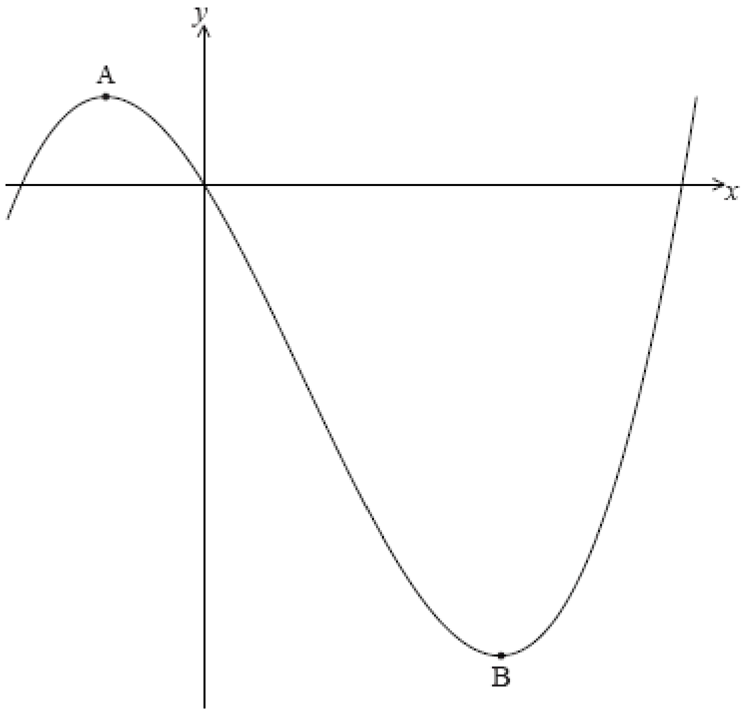
(b) Find the value of

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$.

(6)
(Total 7 marks)

11. Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B(3, –9).

(a) Find the coordinates of A.

(8)

(b) Write down the coordinates of

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

(6)
(Total 14 marks)

12. Let $f(x) = kx^4$. The point P(1, k) lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

(Total 6 marks)

13. Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

(Total 7 marks)

14. Let $g(x) = x^3 - 3x^2 - 9x + 5$.

(a) Find the two values of x at which the tangent to the graph of g is horizontal.

(8)

(b) For each of these values, determine whether it is a maximum or a minimum.

(6)
(Total 14 marks)

Paper 2

Calculator Allowed.

15. The following diagram shows triangle ABC.

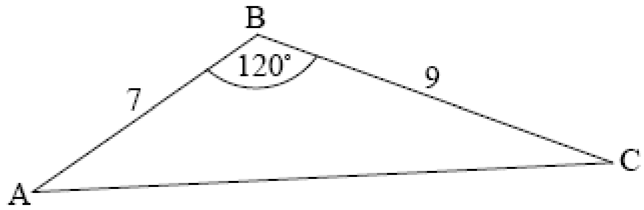


diagram not to scale

AB = 7 cm, BC = 9 cm and $\hat{ABC} = 120°$.

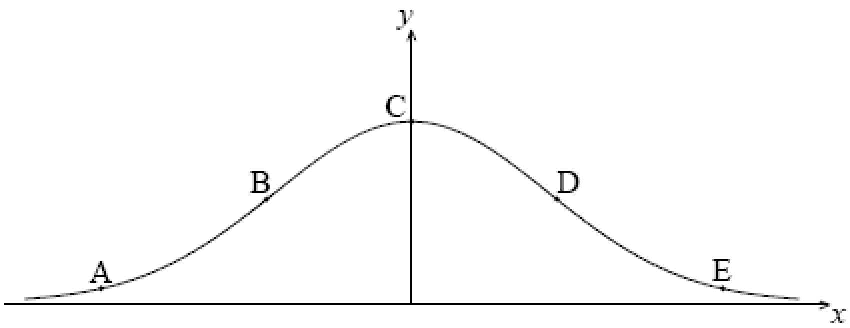
(a) Find AC.

(3)

(b) Find $B\hat{A}C$.

(3)
(Total 6 marks)

16. The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of *f*. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion.

(2)

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)\,e^{-x^2}$.

(5)

(c) Find the x-coordinate of each point of inflexion.

(4)

(d) Use the second derivative to show that one of these points is a point of inflexion.

(4)
(Total 15 marks)

17. The diagram below shows a quadrilateral ABCD with obtuse angles $A\hat{B}C$ and $A\hat{D}C$.

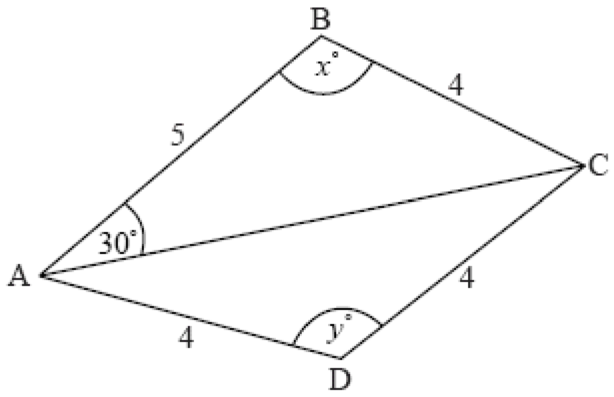


diagram not to scale

AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm, $B\hat{A}C = 30°$, $A\hat{B}C = x°$, $A\hat{D}C = y°$.

(a) Use the cosine rule to show that AC = $\sqrt{41 - 40\cos x}$.

(1)

(b) Use the sine rule in triangle ABC to find another expression for AC.

(2)

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC.

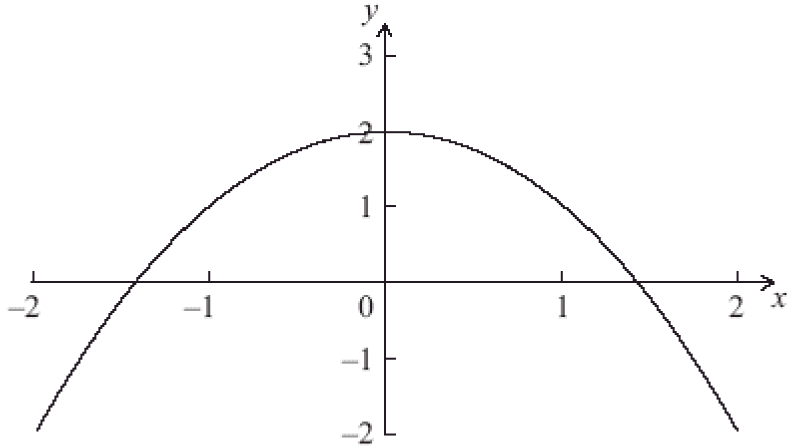
(6)

(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD.

(5)
(Total 14 marks)

18. Consider $f(x) = 2 - x^2$, for $-2 \leq x \leq 2$ and $g(x) = \sin e^x$, for $-2 \leq x \leq 2$. The graph of f is given below.



(a) On the diagram above, sketch the graph of g.

(3)

(b) Solve $f(x) = g(x)$.

(2)

(c) Write down the set of values of x such that $f(x) > g(x)$.

(2)
(Total 7 marks)

19. Solve the equation $e^x = 4 \sin x$, for $0 \leq x \leq 2\pi$.

(Total 5 marks)

20. Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f.

(3)

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

(3)

(c) Hence write $f(x)$ in the form $p\sin(qx + r)$.

(3)

(d) Write down one value of x such that $f'(x) = 0$.

(2)

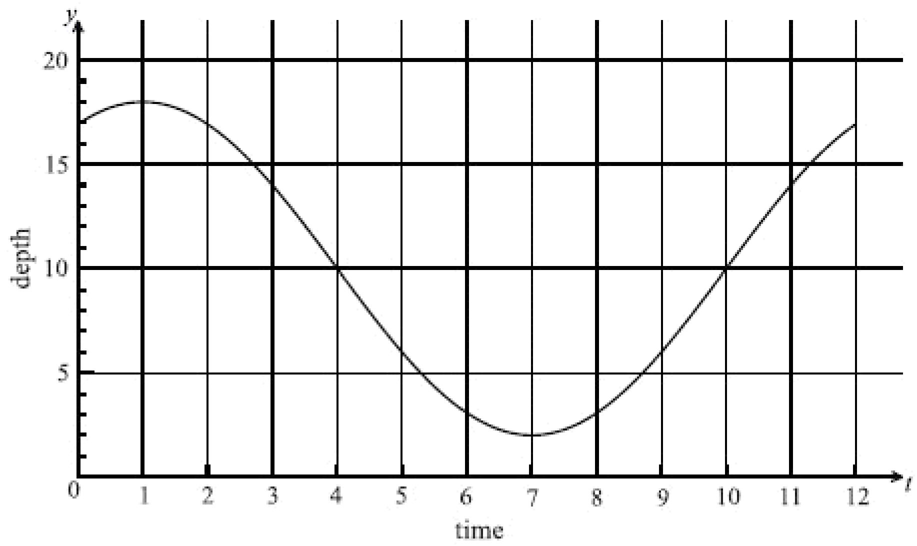
(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions.

(2)

(f) Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x.

(5)
(Total 18 marks)

21. The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

(3)

(b) The depth of water can be modelled by the function $y = A \cos (B (t - 1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B.

(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P.

(2)
(Total 11 marks)

22. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n.$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2560000}{10+90e^{-0.1n}}.$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.

(6)

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

(5)
(Total 17 marks)

23. Let $f(x) = e^{2x} \cos x$, $-1 \leq x \leq 2$.

(a) Show that $f'(x) = e^{2x} (2 \cos x - \sin x)$.

(3)

Let the line L be the normal to the curve of f at $x = 0$.

(b) Find the equation of L.

(5)

The graph of f and the line L intersect at the point (0, 1) and at a second point P.

(c) (i) Find the x-coordinate of P.

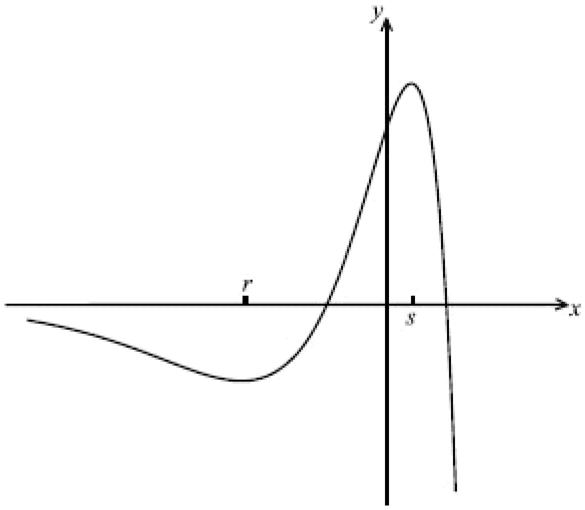
(6)
(Total 14 marks)

24. Let $f(x) = e^x(1 - x^2)$.

(a) Show that $f'(x) = e^x(1 - 2x - x^2)$.

(3)

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote.

(1)

(c) Write down the value of r and of s.

(4)

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$.

(4)

(Total 12 marks)

1.	(a)	attempt to form composite	(M1)		
		e.g. $g(7-2x), 7-2x+3$			
		$(g \circ f)(x) = 10 - 2x$	A1	N2	2
	(b)	$g^{-1}(x) = x - 3$	A1	N1	1
	(c)	**METHOD 1**			
		valid approach	(M1)		
		e.g. $g^{-1}(5)$, 2, $f(5)$			
		$f(2) = 3$	A1	N2	2
		METHOD 2			
		attempt to form composite of f and g^{-1}	(M1)		
		e.g. $(f \circ g^{-1})(x) = 7 - 2(x - 3)$, $13 - 2x$			
		$(f \circ g^{-1})(5) = 3$	A1	N2	2

[5]

2.	(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}, \frac{\mathrm{d}}{\mathrm{d}x}x^2 = 2x$ (seen anywhere)	A1A1		
		attempt to substitute into the quotient rule (do **not** accept product rule)	M1		
		e.g. $\frac{x^2\left(\frac{1}{x}\right)-2x\ln x}{x^4}$			
		correct manipulation that clearly leads to result	A1		
		e.g. $\frac{x-2x\ln x}{x^4}, \frac{x(1-2\ln x)}{x^4}, \frac{x}{x^4}-\frac{2x\ln x}{x^4}$			
		$g'(x) = \frac{1-2\ln x}{x^3}$	AG	N0	4

(b) evidence of setting the derivative equal to zero (M1)

e.g. $g'(x) = 0$, $1 - 2\ln x = 0$

$\ln x = \frac{1}{2}$ A1

$x = e^{\frac{1}{2}}$ A1 N2 3

[7]

3. METHOD 1

using double-angle identity (seen anywhere) A1

e.g. $\sin 2x = 2\sin x \cos x$, $2\cos x = 2\sin x \cos x$

evidence of valid attempt to solve equation (M1)

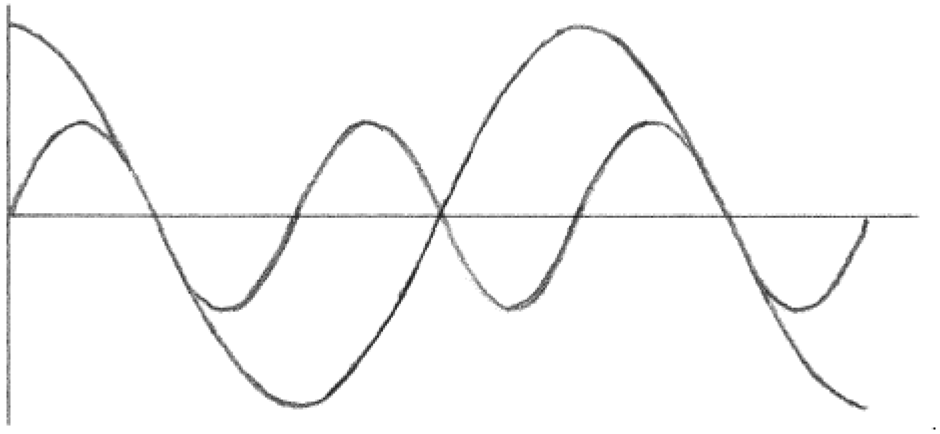
e.g. $0 = 2\sin x \cos x - 2\cos x$, $2\cos x\,(1 - \sin x) = 0$

$\cos x = 0$, $\sin x = 1$ A1A1

$x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x = \frac{5\pi}{2}$ A1A1A1 N4

[7]

METHOD 2



A1A1M1A1

Notes: *Award A1 for sketch of sin 2x, A1 for a sketch of 2 cos x, M1 for at least one intersection point seen, and A1 for 3 approximately correct intersection points. Accept sketches drawn outside [0, 3π], even those with more than 3 intersections.*

$x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x\frac{5\pi}{2}$ A1A1A1 N4

[7]

4. (a) valid approach (M1)

e.g. $b^2 - 4ac, \Delta = 0, (-4k)^2 - 4(2k)(1)$

correct equation A1

e.g. $(-4k)^2 - 4(2k)(1) = 0$, $16k^2 = 8k$, $2k^2 - k = 0$

correct manipulation A1

e.g. $8k(2k-1), \frac{8 \pm \sqrt{64}}{32}$

$k = \frac{1}{2}$ A2 N3 5

(b) recognizing vertex is on the *x*-axis M1

e.g. (1, 0), sketch of parabola opening upward from the *x*-axis

$P \geq 0$ A1 N1 2

[7]

5.	(a)	attempt to find d	(M1)		
		e.g. $\frac{u_3 - u_1}{2}$, $8 = 2 + 2d$			
		$d = 3$	A1	N2	2
	(b)	correct substitution	(A1)		
		e.g. $u_{20} = 2 + (20 - 1)3$, $u_{20} = 3 \times 20 - 1$			
		$u_{20} = 59$	A1	N2	2
	(c)	correct substitution	(A1)		
		e.g. $S_{20} = \frac{20}{2}(2 + 59)$, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$			
		$S_{20} = 610$	A1	N2	2

[6]

6.	(a)	attempt to apply rules of logarithms	(M1)		
		e.g. $\ln a^b = b\ln a$, $\ln ab = \ln a + \ln b$			
		correct application of $\ln a^b = b\ln a$ (seen anywhere)	A1		
		e.g. $3\ln x = \ln x^3$			
		correct application of $\ln ab = \ln a + \ln b$ (seen anywhere)	A1		
		e.g. $\ln 5x^3 = \ln 5 + \ln x^3$			
		so $\ln 5x^3 = \ln 5 + 3\ln x$			
		$g(x) = f(x) + \ln 5$ (accept $g(x) = 3\ln x + \ln 5$)	A1	N1	4
	(b)	transformation with correct name, direction, and value	A3		
		e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by ln 5, vertical translation of ln 5			3

[7]

7.	(a)	(i) 100 (metres)	A1	N1	
		(ii) 50 (metres)	A1	N1	2

(b) (i) identifying symmetry with $h(2) = 9.5$ (M1)

subtraction A1

e.g. $100 - h(2)$, $100 - 9.5$

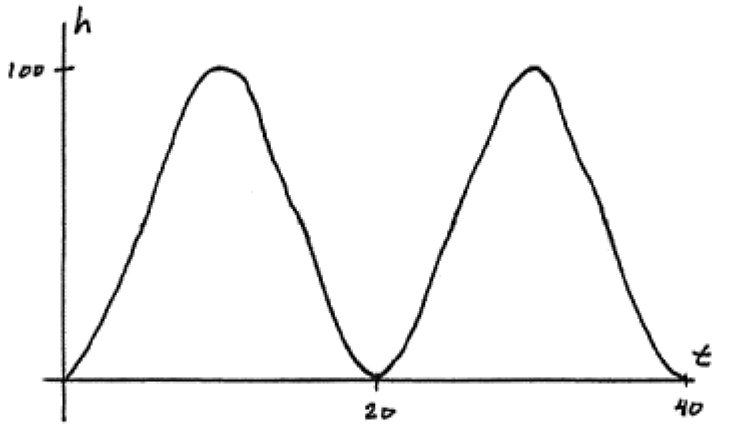
$h(8) = 90.5$ AG N0

(ii) recognizing period (M1)

e.g. $h(21) = h(1)$

$h(21) = 2.4$ A1 N2 4

(c)



A1A1A1 N3 3

Note: *Award A1 for end points (0, 0)* ***and*** *(40, 0), A1 for range $0 \le h \le 100$, A1 for approximately correct sinusoidal shape, with two cycles*

(d) evidence of a quotient involving 20, 2π or 360º to find b (M1)

e.g. $\frac{2\pi}{b} = 20, b = \frac{360}{20}$

$b = \frac{2\pi}{20}\left(= \frac{\pi}{10}\right)$

(accept $b = 18$ if working in degrees) A1 N2

$a = -50$, $c = 50$ A2A1 N3 5

[14]

8. (a) evidence of setting function to zero (M1)
e.g. $f(x) = 0$, $8x = 2x^2$

evidence of correct working A1

e.g. $0 = 2x(4 - x)$, $\frac{-8 \pm \sqrt{64}}{-4}$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0), or $x = 4$, $x = 0$) A1A1 N1N1

(b) (i) $x = 2$ (must be equation) A1 N1

(ii) substituting $x = 2$ into $f(x)$ (M1)
$y = 8$ A1 N2

[7]

10. (a) $\tan\theta = \frac{3}{4}\left(\text{do not accept } \frac{3}{4}x\right)$ A1 N1

(b) (i) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ (A1)(A1)

correct substitution A1

e.g. $\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ A1 N3

(ii) correct substitution A1

e.g. $\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2, \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ A1 N1

[7]

11. (a) $f'(x) = x^2 - 2x - 3$ A1A1A1

evidence of solving $f'(x) = 0$ (M1)

e.g. $x^2 - 2x - 3 = 0$

evidence of correct working A1

e.g. $(x + 1)(x - 3)$, $\frac{2 \pm \sqrt{16}}{2}$

$x = -1$ (ignore $x = 3$) (A1)

evidence of substituting **their negative** x-value into $f(x)$ (M1)

e.g. $\frac{1}{3}(-1)^3 - (-1)^2 - 3(-1)$, $-\frac{1}{3} - 1 + 3$

$y = \frac{5}{3}$ A1

coordinates are $\left(-1, \frac{5}{3}\right)$ N3

(b) (i) $(-3, -9)$ A1 N1

(ii) $(1, -4)$ A1A1 N2

(iii) reflection gives $(3, 9)$ (A1)

stretch gives $\left(\frac{3}{2}, 9\right)$ A1A1 N3

[14]

12. gradient of tangent = 8 (seen anywhere) (A1)
$f'(x) = 4kx^3$ (seen anywhere) A1

recognizing the gradient of the tangent is the derivative (M1)

setting the derivative equal to 8 (A1)
e.g. $4kx^3 = 8$, $kx^3 = 2$

substituting $x = 1$ (seen anywhere) (M1)
$k = 2$ A1 N4

[6]

13. recognizing $\log a + \log b = \log ab$ (seen anywhere) (A1)
e.g. $\log_2(x(x-2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ (seen anywhere) (A1)
e.g. $2^3 = 8$

correct simplification A1
e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve (M1)
e.g. factorizing, quadratic formula

correct working A1

e.g. $(x-4)(x+2)$, $\dfrac{2 \pm \sqrt{36}}{2}$

$x = 4$ A2 N3

[7]

14. (a) Attempt to differentiate (M1)
$g'(x) = 3x^2 - 6x - 9$ A1A1A1

for setting derivative equal to zero M1
$3x^2 - 6x - 9 = 0$

Solving A1
e.g. $3(x-3)(x+1) = 0$
$x = 3$ $x = -1$ A1A1 N3

(b) **METHOD 1**
$g'(x < -1)$ is positive, $g'(x > -1)$ is negative A1A1
$g'(x < 3)$ is negative, $g'(x > 3)$ is positive A1A1

min when $x = 3$, max when $x = -1$ A1A1 N2

METHOD 2

Evidence of using second derivative (M1)

$g''(x) = 6x - 6$ A1

$g''(3) = 12$ (or positive), $g''(-1) = -12$ (or negative) A1A1

min when $x = 3$, max when $x = -1$ A1A1 N2

[14]

15. (a) evidence of choosing cosine rule (M1)

e.g. $a^2 + b^2 - 2ab\cos C$

correct substitution A1

e.g. $7^2 + 9^2 - 2(7)(9)\cos 120^{\circ}$

AC = 13.9 (= $\sqrt{193}$) A1 N2 3

(b) **METHOD 1**

evidence of choosing sine rule (M1)

e.g. $\frac{\sin \hat{A}}{\text{BC}} = \frac{\sin \hat{B}}{\text{AC}}$

correct substitution A1

e.g. $\frac{\sin \hat{A}}{9} = \frac{\sin 120}{13.9}$

$\hat{A} = 34.1^{\circ}$ A1 N2 3

METHOD 2

evidence of choosing cosine rule (M1)

e.g. $\cos \hat{A} = \frac{\text{AB}^2 + \text{AC}^2 - \text{BC}^2}{2(\text{AB})(\text{AC})}$

correct substitution A1

e.g. $\cos \hat{A} = \frac{7^2 + 13.9^2 - 9^2}{2(7)(13.9)}$

$\hat{A} = 34.1^{\circ}$ □ A1 N2 3

[6]

16. (a) B, D A1A1 N2 2

(b) (i) $f'(x) = -2xe^{-x^2}$ A1A1 N2

Note: *Award A1 for* e^{-x^2} *and A1 for –2x.*

	(ii)	finding the derivative of $-2x$, *i.e.* -2	(A1)		
		evidence of choosing the product rule	(M1)		
		e.g. $-2\mathrm{e}^{-x^2} - 2x \times -2x\mathrm{e}^{-x^2}$			
		$-2\mathrm{e}^{-x^2} + 4x^2\mathrm{e}^{-x^2}$	A1		
		$f''(x) = (4x^2 - 2)\,\mathrm{e}^{-x^2}$	AG	N0	5
(c)		valid reasoning	R1		
		e.g. $f''(x) = 0$			
		attempting to solve the equation	(M1)		
		e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$			
		$p = 0.707\left(= \frac{1}{\sqrt{2}}\right)$, $q = -0.707\ \left(= -\frac{1}{\sqrt{2}}\right)$	A1A1	N3	4
(d)		evidence of using second derivative to test values on either side of POI	M1		
		e.g. finding values, reference to graph of f'', sign table			
		correct working	A1A1		
		e.g. finding any two correct values either side of POI,			
		checking sign of f'' on either side of POI			
		reference to sign change of $f''(x)$	R1	N0	4

[15]

17. (a)	correct substitution	A1		
	e.g. $25 + 16 - 40\cos x$, $5^2 + 4^2 - 2 \times 4 \times 5\cos x$			
	$\text{AC} = \sqrt{41 - 40\cos x}$	AG		
(b)	correct substitution	A1		
	e.g. $\frac{\text{AC}}{\sin x} = \frac{4}{\sin 30}, \frac{1}{2}\text{AC} = 4\sin x$			
	$\text{AC} = 8\sin x \left(\text{accept } \frac{4\sin x}{\sin 30}\right)$	A1	N1	

(c) (i) evidence of appropriate approach using AC — M1

e.g. $8 \sin x = \sqrt{41 - 40\cos x}$, sketch showing intersection

correct solution 8.682..., 111.317... — (A1)
obtuse value 111.317... — (A1)

$x = 111.32$ to 2 dp (do **not** accept the radian answer 1.94) — A1 N2

(ii) substituting value of x into either expression for AC — (M1)
e.g. AC = 8 sin 111.32
AC = 7.45 — A1 N2

(d) (i) evidence of choosing cosine rule — (M1)

e.g. $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

correct substitution — A1

e.g. $\dfrac{4^2 + 4^2 - 7.45^2}{2 \times 4 \times 4}$, $7.45^2 = 32 - 32\cos y$, $\cos y = -0.734...$

$y = 137$ — A1 N2

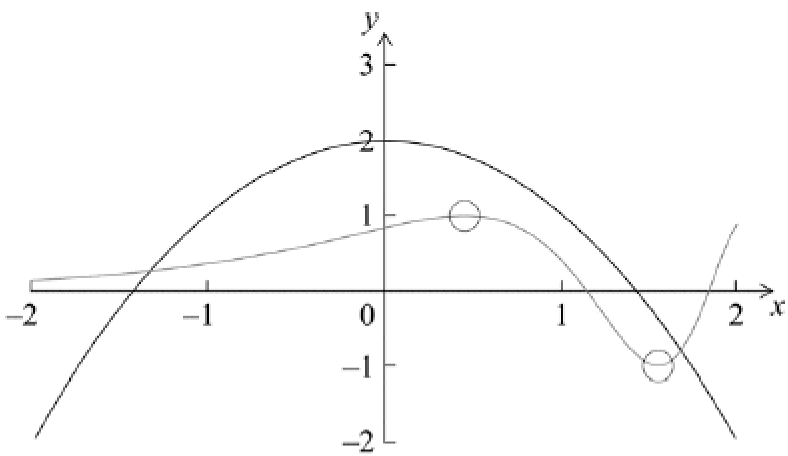
(ii) correct substitution into area formula — (A1)

e.g. $\dfrac{1}{2} \times 4 \times 4 \times \sin 137$, $8 \sin 137$

area = 5.42 — A1 N2

[14]

18. (a)



A1A1A1 N3

(b) $x = -1.32$, $x = 1.68$ (accept $x = -1.41$, $x = 1.39$ if working in degrees) — A1A1 N2

(c) $-1.32 < x < 1.68$ (accept $-1.41 < x < 1.39$ if working in degrees) A2 N2

[7]

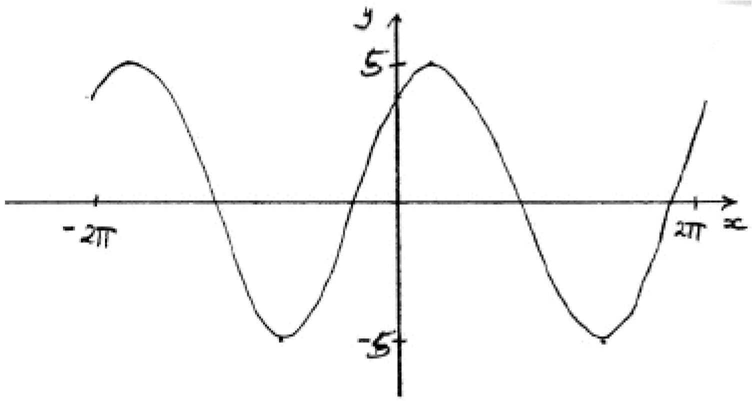
19. evidence of appropriate approach M1

e.g. a sketch, writing $e^x - 4\sin x = 0$

$x = 0.371$, $x = 1.36$ A2A2 N2N2

[5]

20. (a)



A1A1A1 N3

Note: *Award A1 for approximately sinusoidal shape, A1 for end points approximately correct, (–2π, 4), (2π, 4) A1 for approximately correct position of graph, (y-intercept (0, 4) maximum to right of y-axis).*

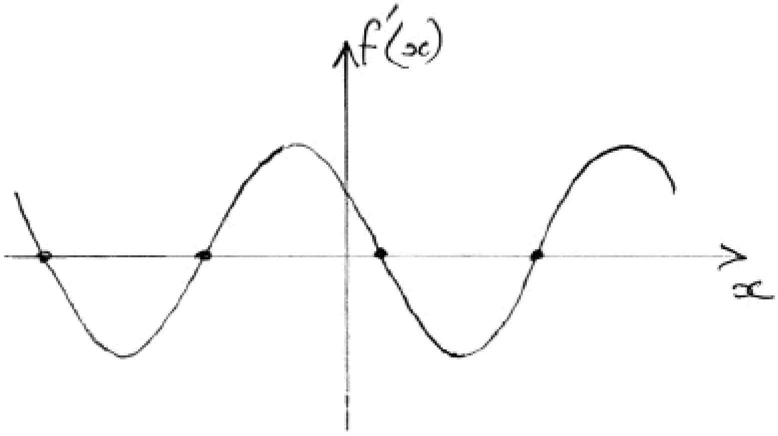
(b) (i) 5 A1 N1

(ii) 2π (6.28) A1 N1

(iii) –0.927 A1 N1

(c) $f(x) = 5\sin(x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) A1A1A1 N3

(d) evidence of correct approach (M1)
e.g. max/min, sketch of $f'(x)$ indicating roots

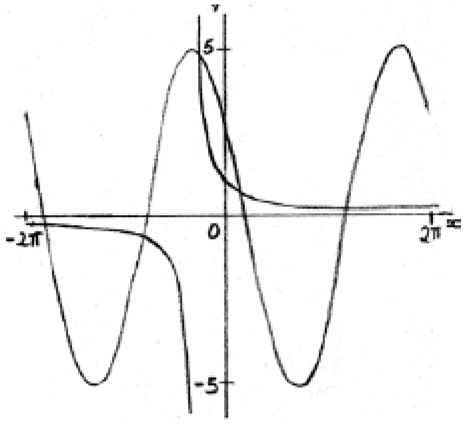


one 3 s.f. value which rounds to one of –5.6, –2.5, 0.64, 3.8 A1 N2

(e) $k = -5$, $k = 5$ A1A1 N2

(f) **METHOD 1**

graphical approach (but must involve derivative functions) M1
e.g.



each curve A1A1
$x = 0.511$ A2 N2

METHOD 2

$g'(x) = \dfrac{1}{x+1}$ A1
$f'(x) = 3\cos x - 4\sin x \quad (5\cos(x + 0.927))$ A1
evidence of attempt to solve $g'(x) = f'(x)$ M1
$x = 0.511$ A2 N2

[18]

21.	(a)	(i)	7	A1	N1
		(ii)	1	A1	N1
		(iii)	10	A1	N1
	(b)	(i)	evidence of appropriate approach	M1	
			e.g. $A=\frac{18-2}{2}$		
			$A = 8$	AG	N0
		(ii)	$C = 10$	A2	N2
		(iii)	**METHOD 1**		
			period = 12	(A1)	
			evidence of using $B \times$ period $= 2\pi$ (accept 360°)	(M1)	
			e.g. $12 = \frac{2\pi}{B}$		
			$B=\frac{\pi}{6}$ (accept 0.524 or 30)	A1	N3
			METHOD 2		
			evidence of substituting *e.g.* $10 = 8\cos 3B + 10$	(M1)	
			simplifying	(A1)	
			e.g. $\cos 3B = 0$ $\left(3B=\frac{\pi}{2}\right)$		
			$B=\frac{\pi}{6}$ (accept 0.524 or 30)	A1	N3
	(c)		correct answers	A1A1	
			e.g. $t = 3.52$, $t = 10.5$, between 03:31 and 10:29 (accept 10:30)		N2

[11]

22.	(a)	(i)	$n = 5$	(A1)	
			$T = 280 \times 1.12^5$		
			$T = 493$	A1	N2
		(ii)	evidence of doubling	(A1)	
			e.g. 560		
			setting up equation	A1	
			e.g. $280 \times 1.12^n = 560$, $1.12^n = 2$		
			$n = 6.116...$	(A1)	
			in the year 2007	A1	N3
	(b)	(i)	$P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(5)}}$	(A1)	
			$P = 39\,635.993...$	(A1)	
			$P = 39\,636$	A1	N3
		(ii)	$P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(7)}}$		
			$P = 46\,806.997...$	A1	
			not doubled	A1	N0
			valid reason for **their** answer	R1	
			e.g. $P < 51200$		
	(c)	(i)	correct value	A2	N2
			e.g. $\dfrac{25600}{280}, 91.4, 640:7$		
		(ii)	setting up an inequality (accept an equation, or reversed inequality)	M1	
			e.g. $\dfrac{P}{T} < 70$, $\dfrac{2\,560\,000}{\left(10 + 90\mathrm{e}^{-0.1n}\right)280 \times 1.12^n} < 70$		
			finding the value 9.31....	(A1)	
			after 10 years	A1	N2

[17]

23.	(a)	correctly finding the derivative of e^{2x}, *i.e.* $2\mathrm{e}^{2x}$	A1	
		correctly finding the derivative of $\cos x$, *i.e.* $-\sin x$	A1	
		evidence of using the product rule, seen anywhere	M1	
		e.g. $f'(x) = 2\mathrm{e}^{2x}\cos x - \mathrm{e}^{2x}\sin x$		
		$f'(x) = \mathrm{e}^{2x}(2\cos x - \sin x)$	AG	N0
	(b)	evidence of finding $f(0) = 1$, seen anywhere	A1	

attempt to find the gradient of f (M1)
e.g. substituting $x = 0$ into $f'(x)$

value of the gradient of f A1
e.g. $f'(0) = 2$, equation of tangent is $y = 2x + 1$

gradient of normal = $-\frac{1}{2}$ (A1)

$y - 1 = -\frac{1}{2}x \quad \left(y = -\frac{1}{2}x + 1\right)$ A1 N3

(c) (i) evidence of equating correct functions M1
e.g. $e^{2x}\cos x = -\frac{1}{2}x + 1$, sketch showing intersection of graphs
$x = 1.56$ A1 N1

[14]

24. (a) evidence of using the product rule M1

$f'(x) = e^x(1 - x^2) + e^x(-2x)$ A1A1

Note: *Award A1 for* $e^x(1-x^2)$, *A1 for* $e^x(-2x)$.

$f'(x) = e^x(1 - 2x - x^2)$ AG N0

(b) $y = 0$ A1 N1

(c) at the local maximum or minimum point

$f'(x) = 0 \quad (e^x(1 - 2x - x^2) = 0)$ (M1)

$\Rightarrow 1 - 2x - x^2 = 0$ (M1)
$r = -2.41$ $s = 0.414$ A1A1 N2N2

(d) $f'(0) = 1$ A1
gradient of the normal = –1 A1
evidence of substituting into an equation for a straight line (M1)
correct substitution A1
e.g. $y - 1 = -1(x - 0)$, $y - 1 = -x$, $y = -x + 1$
$x + y = 1$ AG N0

(e) (i) intersection points at $x = 0$ and $x = 1$ (A1)