

Summer Packet P1 [39 marks]

In this question, give all answers to two decimal places.

Karl invests 1000 US dollars (USD) in an account that pays a nominal annual interest of 3.5%, **compounded quarterly**. He leaves the money in the account for 5 years.

1a. Calculate the amount of money he has in the account after 5 years. [3 marks]

1b. Write down the amount of **interest** he earned after 5 years. [1 mark]

1c. Karl decides to donate this **interest** to a charity in France. The charity receives 170 euros (EUR). The exchange rate is 1 USD = t EUR.

Calculate the value of t .

2a. Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$, where $n \in \mathbb{Z}$. [2 marks]

2b. Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. [3 marks]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

3a. Show that $(g \circ f)(x) = 2x + 11$. [2 marks]

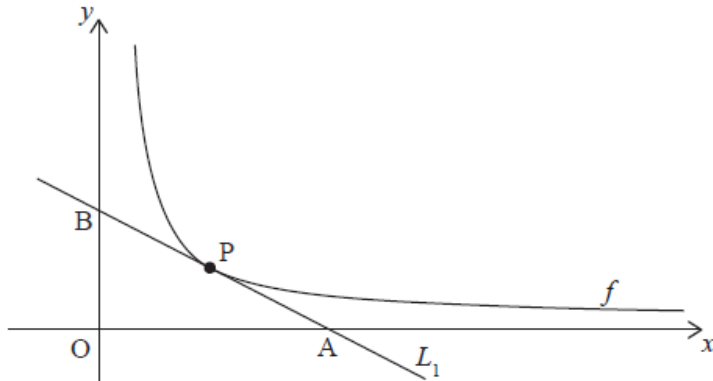
3b. Given that $(g \circ f)^{-1}(a) = 4$, find the value of a . [3 marks]

4a. Show that $\log_9(\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$. [3 marks]

4b. Hence or otherwise solve $\log_3(2 \sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. [5 marks]

The following diagram shows part of the graph of $f(x) = \frac{k}{x}$, for $x > 0, k > 0$.

Let $P\left(p, \frac{k}{p}\right)$ be any point on the graph of f . Line L_1 is the tangent to the graph of f at P .



5a. Find $f'(p)$ in terms of k and p .

[2 marks]

5b. Show that the equation of L_1 is $kx + p^2y - 2pk = 0$.

[2 marks]

Line L_1 intersects the x -axis at point $A(2p, 0)$ and the y -axis at point B .

5c. Find the area of triangle AOB in terms of k .

[5 marks]

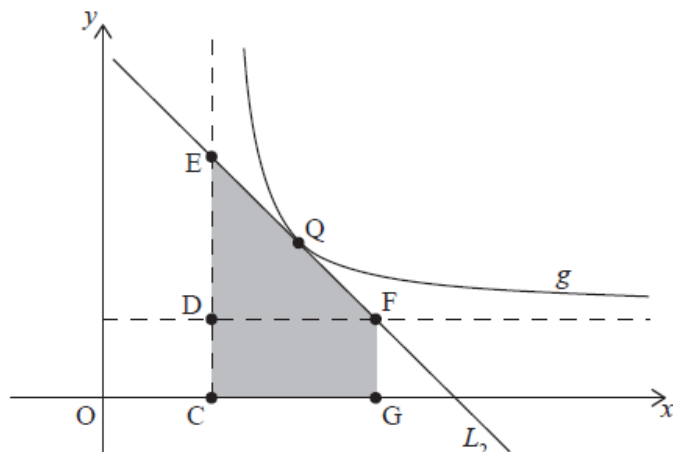
5d. The graph of f is translated by $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ to give the graph of g .

[6 marks]

In the following diagram:

- point Q lies on the graph of g
- points C , D and E lie on the vertical asymptote of g
- points D and F lie on the horizontal asymptote of g
- point G lies on the x -axis such that FG is parallel to DC .

Line L_2 is the tangent to the graph of g at Q , and passes through E and F .



Given that triangle EDF and rectangle $CDFG$ have equal areas, find the gradient of L_2 in terms of p .