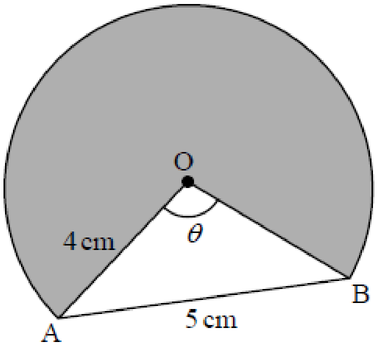
Summer Packet Paper 2 *[64 marks]*

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and AÔB = θ.

1a. Find the value of θ, giving your answer in radians. *[3 marks]*

1b. Find the area of the shaded region. *[3 marks]*

On 1st January 2020, Laurie invests \$$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

2a. Find the value of r, giving your answer to four significant figures. *[3 marks]*

2b. Laurie makes no further deposits to or withdrawals from the account. *[3 marks]*

Find the year in which the amount of money in Laurie's account will become double the amount she invested.

Consider a function f, such that $f\left(x\right) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

3a. Find the period of f. *[2 marks]*

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

3b. Find the value of b. *[2 marks]*

3c. Hence, find the value of $f(6)$. *[2 marks]*

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q$, 0 ≤ x ≤ 10; p, $q \in \mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

3d. Find the value of p and the value of q. *[5 marks]*

3e. Find the value of x for which the functions have the greatest difference. *[2 marks]*

Let $f(x)= 4 - x^3$ and $g(x)= \ln x$, for $x > 0$.

4a. Find $(f \circ g)(x)$. *[2 marks]*

4b. Solve the equation $(f \circ g)(x)= x$. *[2 marks]*

4c. Hence or otherwise, given that $g(2a)= f^{-1}(2a)$, find the value of a. *[3 marks]*

OAB is a sector of the circle with centre O and radius r, as shown in the following diagram.

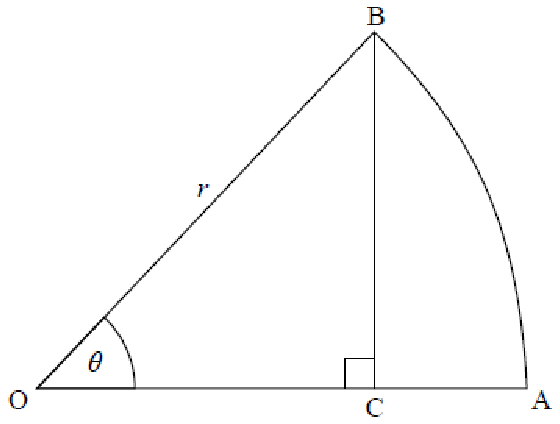


diagram not to scale

The angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$.

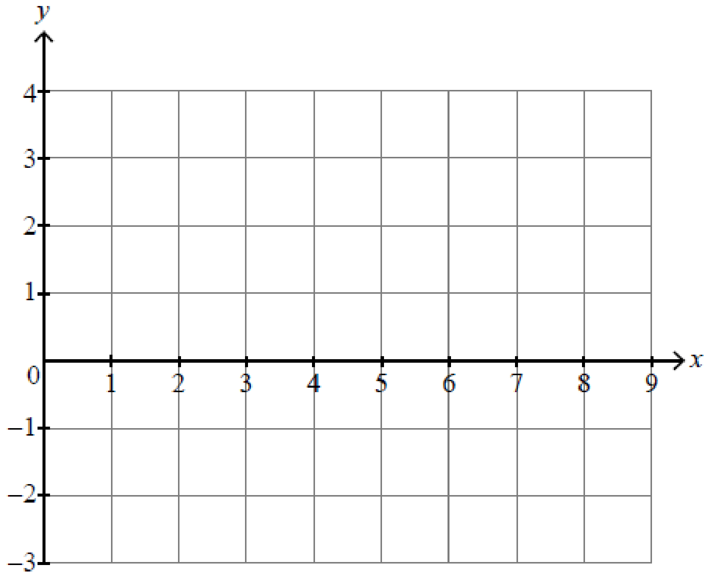
The point C lies on OA and OA is perpendicular to BC.

5a. Show that $\mathrm{OC} = r\cos\theta$. *[1 mark]*

5b. Find the area of triangle OBC in terms of r and θ. *[2 marks]*

5c. Given that the area of triangle OBC is $\frac{3}{5}$ of the area of sector OAB, find θ. *[4 marks]*

6a. Sketch the graphs $y = \sin^3 x + \ln x$ and $y = 1 + \cos x$ on the following axes for $0 < x \leq 9$. *[2 marks]*



6b. Hence solve $\sin^3 x + \ln x - \cos x - 1 < 0$ in the range $0 < x \leq 9$. *[4 marks]*

Let $f(x) = \ln x - 5x$, for $x > 0$.

7. Solve $f'(x) = f''(x)$. *[2 marks]*

Consider the function $f(x) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is a constant.

The graph of the function passes through the point with coordinates (4 , 2).

P is the minimum point of the graph of $f(x)$.

8. Sketch the graph of $y = f(x)$ for $0 < x \leq 6$ and $-30 \leq y \leq 60$. *[4 marks]*
Clearly indicate the minimum point P and the x-intercepts on your graph.

Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$

9a. Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$. *[4 marks]*

9b. A teacher asks her students to make some observations about the curve. *[1 mark]*

Three students responded.
Nadia said *"The x-intercept of the curve is between −1 and zero".*
Rick said *"The curve is decreasing when x < 1 ".*
Paula said *"The gradient of the curve is less than zero between x = 1 and x = 2 ".*

State the name of the student who made an **incorrect** observation.

9c. Find $\frac{\mathrm{dy}}{\mathrm{dx}}$. *[3 marks]*

9d. Show that the stationary points of the curve are at $x = 1$ and $x = 2$. *[2 marks]*

9e. Given that $y = 2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k. *[3 marks]*
