

# Summer Review IB MAI SL

## entering Year 2 *[204 marks]*

Jae Hee plays a game involving a biased six-sided die.

The faces of the die are labelled  $-3$ ,  $-1$ ,  $0$ ,  $1$ ,  $2$  and  $5$ .

The score for the game,  $X$ , is the number which lands face up after the die is rolled.

The following table shows the probability distribution for  $X$ .

<b>Score <math>x</math></b>	$-3$	$-1$	$0$	$1$	$2$	$5$
<b><math>P(X=x)</math></b>	$\frac{1}{18}$	$p$	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

1a. Find the exact value of  $p$ .

*[1 mark]*

Jae Hee plays the game once.

1b. Calculate the expected score.

*[2 marks]*

1c. Jae Hee plays the game twice and adds the two scores together.

*[3 marks]*

Find the probability Jae Hee has a **total** score of  $-3$ .

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

2a. Find the probability that on any given day Mr Burke chooses a female student to answer a question.

*[1 mark]*

In the first month, Mr Burke will teach his class 20 times.

2b. Find the probability he will choose a female student 8 times.

*[2 marks]*

2c. Find the probability he will choose a male student at most 9 times. [3 marks]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

<b>Number of classes in which the students used the internet</b>	0	1	2	3	4	5	6
<b>Number of students</b>	20	24	30	$k$	10	3	1

3a. State whether the data is discrete or continuous. [1 mark]

The mean number of classes in which a student used the internet is 2.

3b. Find the value of  $k$ . [4 marks]

3c. It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list. [1 mark]

Identify the sampling technique used in the survey.

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population,  $P$  could be modelled by  $P = 1350 + 400(1.25)^{-t}$ ,  $t \geq 0$ , where  $t$  is the number of days since the start of the migration season.

4a. Find the population of the Bulbul birds at the start of the migration season. [1 mark]

4b. Find the population of the Bulbul birds after 5 days. [2 marks]

4c. Calculate the time taken for the population to decrease below 1400. [2 marks]

4d. According to this model, find the smallest possible population of Bulbul birds during the migration season. [1 mark]

The intensity level of sound,  $L$  measured in decibels (dB), is a function of the sound intensity,  $S$  watts per square metre ( $\text{W m}^{-2}$ ). The intensity level is given by the following formula.

$$L = 10 \log_{10} (S \times 10^{12}), S \geq 0.$$

5a. An orchestra has a sound intensity of  $6.4 \times 10^{-3} \text{W m}^{-2}$ . Calculate the intensity level,  $L$  of the orchestra. [2 marks]

5b. A rock concert has an intensity level of 112 dB. Find the sound intensity,  $S$ . [2 marks]

**In this question, give all answers to two decimal places.**

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

**Finance option A:**

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

6a. Find the repayment made each quarter. [3 marks]

6b. Find the total amount paid for the car. [2 marks]

6c. Find the interest paid on the loan. [2 marks]

**Finance option B:**

A 6 year loan at a nominal annual interest rate of  $r$  % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

6d. Find the amount to be borrowed for this option. [2 marks]

6e. Find the annual interest rate,  $r$ . [3 marks]

6f. State which option Bryan should choose. Justify your answer. [2 marks]

6g. Bryan's car depreciates at an annual rate of 25 % per year. Find the value of Bryan's car six years after it is purchased. [3 marks]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score ( $x$ )	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score ( $y$ )	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

7a. Write down the value of the Pearson's product-moment correlation coefficient,  $r$ . *[2 marks]*

7b. Using the value of  $r$ , interpret the relationship between Stan's score and Minsun's score. *[2 marks]*

7c. Write down the equation of the regression line  $y$  on  $x$ . *[2 marks]*

7d. Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10. *[2 marks]*

7e. State whether this estimate is reliable. Justify your answer. *[2 marks]*

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

7f. **Copy** and complete the information in the following table. *[2 marks]*

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

7g. Find the value of the Spearman's rank correlation coefficient,  $r_s$ . *[2 marks]*

7h. Comment on the result obtained for  $r_s$ . *[2 marks]*

7i. The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1. *[1 mark]*

Explain why the value of the Spearman's rank correlation coefficient  $r_s$  does not change.

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed,  $s \text{ m s}^{-1}$ , and braking distance,  $d \text{ m}$ , of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ m s}^{-1}$	0	6	10
Braking distance, $d \text{ m}$	0	12	60

This information was used to create Model A, where  $d$  is a function of  $s$ ,  $s \geq 0$ .

Model A:  $d(s) = ps^2 + qs$ , where  $p, q \in \mathbb{Z}$

At a speed of  $6 \text{ m s}^{-1}$ , Model A can be represented by the equation  $6p + q = 2$ .

8a. Write down a second equation to represent Model A, when the speed is  $10 \text{ m s}^{-1}$ . [2 marks]

8b. Find the values of  $p$  and  $q$ . [2 marks]

8c. Find the coordinates of the vertex of the graph of  $y = d(s)$ . [2 marks]

8d. Using the values in the table and your answer to part (b), sketch the graph of  $y = d(s)$  for  $0 \leq s \leq 10$  and  $-10 \leq d \leq 60$ , clearly showing the vertex. [3 marks]

8e. Hence, identify why Model A may not be appropriate at lower speeds. [1 mark]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B:  $d(s) = 0.95s^2 - 3.92s$

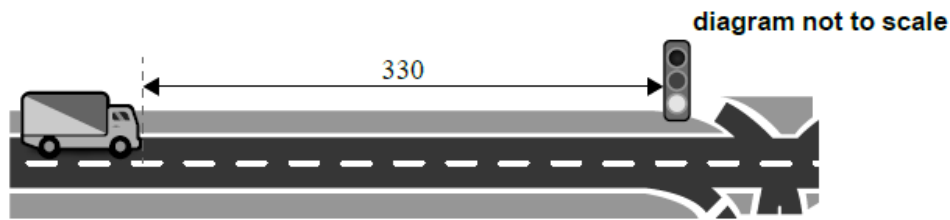
8f. Use Model B to calculate an estimate for the braking distance at a speed of  $20 \text{ m s}^{-1}$ . [2 marks]

The actual braking distance at  $20 \text{ m s}^{-1}$  is 320 m.

8g. Calculate the percentage error in the estimate in part (e). [2 marks]

8h. It is found that once a driver realizes the need to stop their vehicle, 1.6 [3 marks] seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed  $s \text{ m s}^{-1}$ . The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of 330 m.



Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection.

*Give your answers to this question correct to two decimal places.*

Gen invests \$2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

9a. Calculate the value of her savings after 10 years. [2 marks]

9b. The rate of inflation during this 10 year period is 1.5% per year. [3 marks]  
Calculate the real value of her savings after 10 years.

A set of data comprises of five numbers  $x_1, x_2, x_3, x_4, x_5$  which have been placed in ascending order.

10a. Recalling definitions, such as the Lower Quartile is the  $\frac{n+1}{4}th$  piece of [2 marks] data with the data placed in order, find an expression for the Interquartile Range.

10b. Hence, show that a data set with only 5 numbers in it cannot have any [5 marks] outliers.

10c. Give an example of a set of data with 7 numbers in it that does have an [2 marks] outlier, justify this fact by stating the Interquartile Range.

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of \$4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

11a. Calculate the amount Yejin needs to have saved into her annuity fund, *[3 marks]* in order to meet her retirement goal.

11b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. *[3 marks]*

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

Paul wants to buy a car. He needs to take out a loan for \$7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay \$200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

12a. the number of months it will take for Paul to repay the loan. *[3 marks]*

12b. the total amount that Paul has to pay. *[2 marks]*

Use option 2 to calculate

12c. the amount Paul pays each month. *[2 marks]*

12d. the total amount that Paul has to pay. *[2 marks]*

Give a reason why Paul might choose

12e. option 1. *[1 mark]*

12f. option 2. *[1 mark]*

Sophie is planning to buy a house. She needs to take out a mortgage for \$120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay \$1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

13a. Calculate the monthly repayment using option 1. *[2 marks]*

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13b. Calculate the total amount Sophie would pay, using option 1. *[2 marks]*

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13c. Calculate the number of months it will take to repay the mortgage using option 2. *[3 marks]*

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13d. Calculate the total amount Sophie would pay, using option 2. *[2 marks]*

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Give a reason why Sophie might choose

13e. option 1. *[1 mark]*

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13f. option 2. *[1 mark]*

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Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

13g. Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years. *[2 marks]*

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13h. Hence calculate her monthly repayment for the final 10 years. *[2 marks]*

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Urvashi wants to model the height of a moving object. She collects the following data showing the height,  $h$  metres, of the object at time  $t$  seconds.

$t$ (seconds)	2	5	7
$h$ (metres)	34	38	24

She believes the height can be modeled by a quadratic function,  $h(t) = at^2 + bt + c$ , where  $a, b, c \in \mathbb{R}$ .

14a. Show that  $4a + 2b + c = 34$ . [1 mark]

14b. Write down two more equations for  $a, b$  and  $c$ . [3 marks]

14c. Solve this system of three equations to find the value of  $a, b$  and  $c$ . [4 marks]

Hence find

14d. when the height of the object is zero. [3 marks]

14e. the maximum height of the object. [2 marks]

Iron in the asteroid *16 Psyche* is said to be valued at 8973 quadrillion euros (EUR), where one quadrillion =  $10^{15}$ .

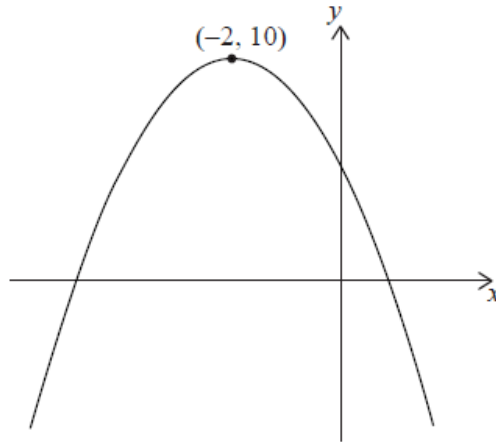
15a. Write down the value of the iron in the form  $a \times 10^k$  where  $1 \leq a < 10, k \in \mathbb{Z}$ . [2 marks]

James believes the asteroid is approximately spherical with radius 113 km. He uses this information to estimate its volume.

15b. Calculate James's estimate of its volume, in  $\text{km}^3$ . [2 marks]

15c. The actual volume of the asteroid is found to be  $6.074 \times 10^6 \text{ km}^3$ . [2 marks]  
Find the percentage error in James's estimate of the volume.

The diagram shows the graph of the quadratic function  $f(x) = ax^2 + bx + c$ , with vertex  $(-2, 10)$ .



The equation  $f(x) = k$  has two solutions. One of these solutions is  $x = 2$ .

16a. Write down the other solution of  $f(x) = k$ . [2 marks]

16b. Complete the table below placing a tick (✓) to show whether the unknown parameters  $a$  and  $b$  are positive, zero or negative. The row for  $c$  has been completed as an example. [2 marks]

	positive	zero	negative
$a$			
$b$			
$c$	✓		

16c. State the values of  $x$  for which  $f(x)$  is decreasing. [2 marks]

Andre will play in the semi-final of a tennis tournament.

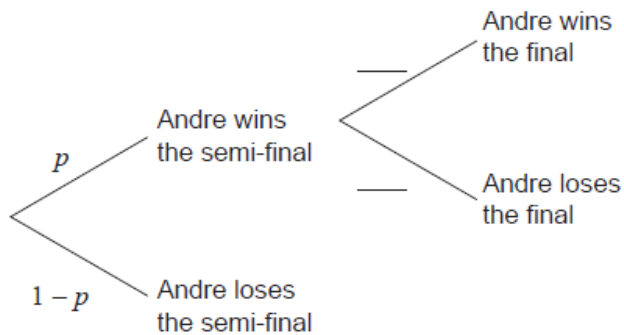
If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is  $p$ . If Andre wins the semi-final, then the probability he will be the champion is 0.6.

17a. Complete the values in the tree diagram.

[1 mark]



The probability that Andre will not be the champion is 0.58.

17b. Find the value of  $p$ .

[2 marks]

17c. Given that Andre did not become the champion, find the probability that he lost in the semi-final. [3 marks]

Olava's Pizza Company supplies and delivers large cheese pizzas.

The total cost to the customer,  $C$ , in Papua New Guinean Kina (PGK), is modelled by the function

$$C(n) = 34.50n + 8.50, n \geq 2, n \in \mathbb{Z},$$

where  $n$ , is the number of large cheese pizzas ordered. This total cost includes a fixed cost for delivery.

18a. State, in the context of the question, what the value of 34.50 represents. [1 mark]

18b. State, in the context of the question, what the value of 8.50 represents. [1 mark]

18c. Write down the minimum number of pizzas that can be ordered.

[1 mark]

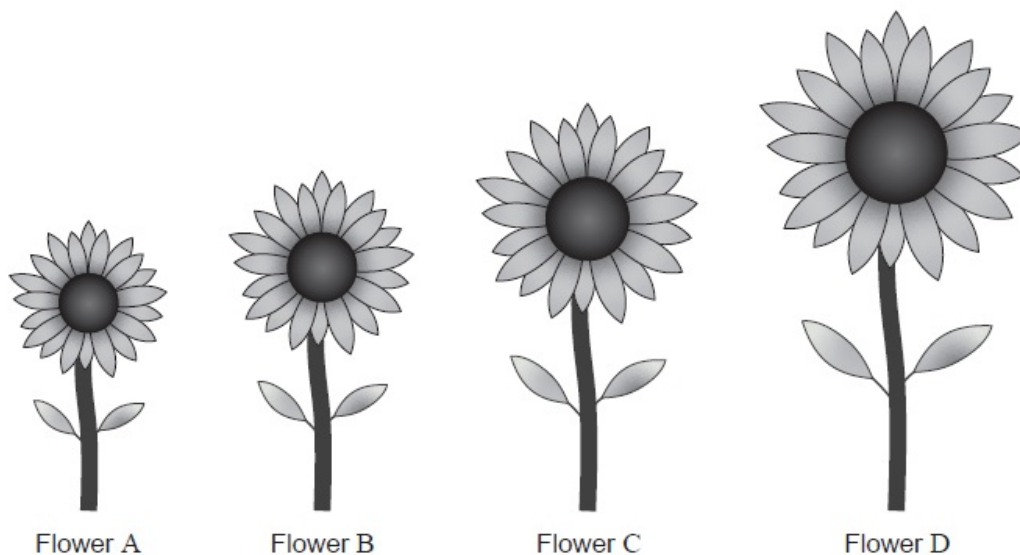
18d. Kaelani has 450 PGK.

[3 marks]

Find the maximum number of large cheese pizzas that Kaelani can order from Olava's Pizza Company.

Anne-Marie planted four sunflowers in order of height, from shortest to tallest.

diagram not to scale



Flower C is 32 cm tall.

The median height of the flowers is 24 cm.

19a. Find the height of Flower null.

[2 marks]

The range of the heights is 50 cm. The height of Flower A is  $p$  cm and the height of Flower D is  $q$  cm.

19b. Using this information, write down an equation in  $p$  and  $q$ .

[1 mark]

The mean height of the flowers is 27 cm.

19c. Write down a second equation in  $p$  and  $q$ .

[1 mark]

19d. Using your answers to **parts (b) and (c)**, find the height of Flower A.

[1 mark]

19e. Using your answers to **parts (b) and (c)**, find the height of Flower D.

[1 mark]

**Give your answers in this question correct to the nearest whole number.**

Imon invested 25 000 Singapore dollars (SGD) in a fixed deposit account with a nominal annual interest rate of 3.6%, compounded **monthly**.

20a. Calculate the value of Imon's investment after 5 years. [3 marks]

20b. At the end of the 5 years, Imon withdrew  $x$  SGD from the fixed deposit account and reinvested this into a super-savings account with a nominal annual interest rate of 5.7%, compounded **half-yearly**. [3 marks]

The value of the super-savings account increased to 20 000 SGD after 18 months. Find the value of  $x$ .

Lucy sells hot chocolate drinks at her snack bar and has noticed that she sells more hot chocolates on cooler days. On six different days, she records the maximum daily temperature,  $T$ , measured in degrees centigrade, and the number of hot chocolates sold,  $H$ . The results are shown in the following table.

Maximum temperature ( $T$ )	14	8	4	18	13	11
Number of hot chocolates ( $H$ )	79	143	191	58	84	105

The relationship between  $H$  and  $T$  can be modelled by the regression line with equation  $H = aT + b$ .

21a. Find the value of  $a$  and of  $b$ . [3 marks]

21b. Write down the correlation coefficient. [1 mark]

21c. Using the regression equation, estimate the number of hot chocolates that Lucy will sell on a day when the maximum temperature is  $12^\circ\text{C}$ . [2 marks]

A discrete random variable  $X$  has the following probability distribution.

$x$	0	1	2	3
$P(X=x)$	$q$	$4p^2$	$p$	$0.7 - 4p^2$

22a. Find an expression for  $q$  in terms of  $p$ . [2 marks]

22b. Find the value of  $p$  which gives the largest value of  $E(X)$ . [3 marks]

22c. Hence, find the largest value of  $E(X)$ .

[1 mark]

Let the universal set,  $U$ , be the set of all integers  $x$  such that  $1 \leq x < 11$ .

$A$ ,  $B$  and  $C$  are subsets of  $U$ .

$$A = \{1, 2, 3, 4, 6, 8\}$$

$$B = \{2, 3, 5, 7\}$$

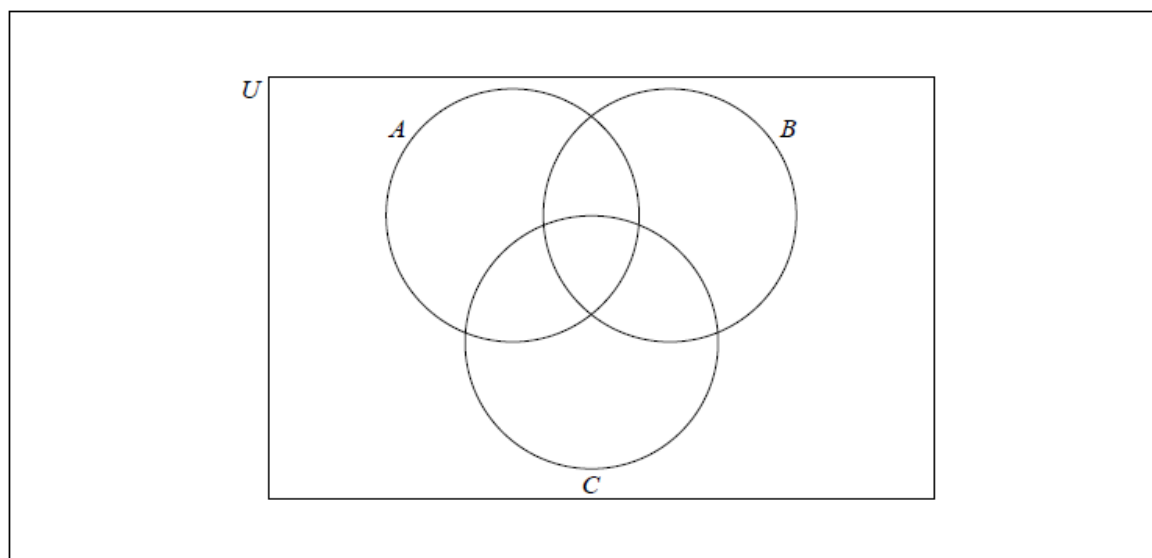
$$C = \{1, 3, 5, 7, 9\}$$

23a. Write down  $n(B)$ .

[1 mark]

23b. Complete the following Venn diagram using **all** elements of  $U$ .

[4 marks]



23c. Write down an element that belongs to  $(A \cup B)' \cap C$ .

[1 mark]

A geometric sequence has a first term of  $\frac{8}{3}$  and a fourth term of 9.

24a. Find the common ratio.

[2 marks]

24b. Write down the second term of this sequence.

[1 mark]

24c. The sum of the first  $k$  terms is greater than 2500.

[3 marks]

Find the smallest possible value of  $k$ .

The depth of water in a port is modelled by the function  $d(t) = p \cos qt + 7.5$ , for  $0 \leq t \leq 12$ , where  $t$  is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

25a. Find the value of  $p$ .

[2 marks]

25b. Find the value of  $q$ .

[2 marks]

25c. Use the model to find the depth of the water 10 hours after high tide. [2 marks]